Trumpet Acoustics: Correcting intonation faults.

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ABSTRACT

Calculations are possible of the locations and sizes of constrictions and enlargements that may be put into a trumpet air column for correcting intonation errors of the trumpet. In certain respects, and to at least a useful approximation, the trumpet behaves as a simple, closed-open, cylindrical resonator, and perturbation theory applied to a closed-open cylinder model permits the calculations of the locations and sizes of the constrictions and enlargements. The corrector-amplitude versus distance function is a Fourier transform of the experimentally-determined intonation-error versus mode function. An interesting experiment, making use of the theory, is the raising of the (usually) flat fifth mode without significantly affecting any of the other playing modes.

PRESENTATION SCRIPT

San Diego, CA, November 19, 1976

INTRODUCTION

Mr. Chairman, Ladies and Gentlemen:

As you know, the abstracts for papers for this meeting had to be submitted in early August. On August 26, an article appeared in the British scientific journal, Nature, on this identical subject, the title being: "Systematic approach to the correction of intonation in wind instruments". The authors were Richard A. Smith and Geoffrey J. Daniell. Now that their paper is available to you in the literature, it seems to me that I can make this presentation most valuable if I relate it to the Smith and Daniell paper, omitting things here that are well disclosed there, and emphasizing what is different.

I shall put in a little introductory material for those who have not delved into the subject before. Anyone who is seriously interested, should certainly study the Smith and Daniell paper.

My personal pleasure at the appearance of their paper has been increased by the privilege of meeting and talking with the author, Richard Smith, in London, last month.

SLIDE 1

A trumpet is represented here without its valves, which are an unnecessary complication for our immediate discussion. (Slide 1 should have the lefthand region of the trumpet labeled as the "mouthpiece plus leaderpipe", or, if a single word, the "mouthpipe").

The lip source of pulsating air, applied to the mouthpiece end, is a relatively high impedance source, so that the trumpet is, effectively, an acoustic resonator closed at one end and open at the other. In spite of this, the vibrational modal frequencies of its air column approximate those of a simple cylindrical resonator open at both ends. This phenomenon has been called "the mode paradox". It has been argued about even in the Journal of the ASA surprisingly recently. The answer is a century old. D. J. Blaikley, who was, interestingly enough, the managing director of Boosey and Hawkes, where Richard Smith is now employed, showed in his 1878 paper that the trumpet is a closed-open resonator whose shape deviations from cylindricality cause its vibrational modes to approximate those of an open-open cylinder.

This is an appropriate point to mention the fundamental difference between the basic assumptions I have used and those of Smith and Daniell.

I have assumed that the trumpet still is a closed-open cylindrical resonator, although a perturbed one. This leads to mathematics like that used by Schroeder and Mermelstein in their 1967 JASA papers on vocal tract shapes. It is Fourier Transform mathematics.

Smith and Daniell begin with a wave equation that ostensibly takes into account the existing cross-sectional variations at the outset. Their assumptions, about the applicability of the wave equations, are less drastic than mine, but also much less obvious, and therefore more likely to remain undiscovered by the reader. A comforting point is that all of us tie our calculations to experiments at the beginning, and in the end, so any intermediate effects of not-completely-valid assumptions can be neutralized.

SLIDE 2

Side 2 shows a schematic diagram of the apparatus I use. It is a variant of apparatus already described in several places in the literature, for instance, in articles by Benade and, particularly, by Backus. It measures the acoustic input impedance of the trumpet at any desired frequency, looking into the trumpet at the mouthpiece end, from a plane a short distance inside the end plane of the mouthpiece. My particular apparatus has a micrometer screw to control the microphone projection distance into the mouthpiece. This is to simulate the varying lip insertions that trumpeters use. I believe this facility is highly desirable.

The apparatus of Smith and Daniell automatically searches for resonance peaks and gives digital printouts of peak frequencies and amplitudes. I would recommend full plots of the resonance peaks, or the equivalent in visual observation of the continuous behavior of an analog voltmeter needle. This is because in my work with perturbers in bores, I have encountered peaks surmounted by peak<u>lets</u> whose relative dominance shifted down in frequency when the main peak body shifted up, and vice versa; so I would be uncomfortable with only digital peak printouts.

SLIDE 3

Slide 3 represents some of the lower modal vibrations in a simple cylindrical resonator, closed at the left end and open at the right end. Inside the sounding trumpet, there are alternate nodes and antinodes in the same numbers as those in a simple cylindrical resonator, but they are not evenly spaced along the axis, particularly when the low modes are being sounded. They tend to become evenly spaced for the upper modes.

Musicians, as well as acousticians, have known for decades that if one makes a bore enlargement in a tubular resonator, the frequencies of all the vibrational modes that have a velocity antinode at the position of that enlargement, will be raised, and the frequencies of all those modes that have a velocity node at that position will be lowered.

For instance if you imagine a bore enlargement at a fractional distance of two-thirds in the resonator represented in Slide 3, it would lower the frequency of the second mode, raise that of the fourth mode, and lower that of the fifth mode.

Constrictions have effects just the opposite of enlargements.

Highly sophisticated derivations of some of the fundamental formulas concerning these effects were given in JASA articles of Schroeder and of Mermelstein in 1967. For the purposes of this presentation it is convenient to think of the effects in terms of simple cosine curves showing the degree of sharping, or flatting, versus mode number for various axial positions of a perturbing constriction (or enlargement), as shown in the next slide.

SLIDE 4

Each curve in Slide 4 shows the sharping, or flatting, effect of a single constriction placed at the fractional distance in the bore that is specified at the right hand side of the curve. The sharping, or flatting is plotted against the mode number. A constriction at a fractional distance of only 1/15 has a monotonic effect throughout the usual playing modes, first sharping the lower modes, then flatting the upper modes, through the eighth.

As the fractional distance is increased, the curves become multi-crested waves. As the fractional position of 1/2 is approached, the wavelength of the mode shift curve approaches only 2 modes, so the effect is to sharp one mode, flat the next, sharp the next, flat the next, and so on.

These simple-theoretical waves are all cosine curves, in phase at the fictitious mode number, 1/2.

Fourier Analysis tells us that we should be able to assemble a set of these curves that would sum to an arbitrarily specified set of flattings and sharpings of the individual modes. One of the interesting questions that can then arise is the following: Anything we do at one place in a bore, to raise or lower a particular mode, affects all the other modes and in ways we may not want. Can we perturb a set of positions in a bore so that the combined effects on a mode that we want to shift will add, and the combined effects on the other modes will cancel? The answer is yes, as shown on the next slide.

SLIDE 5

Here is a beautiful little theorem on the possibility of shifting one mode only. It says that "equal point perturbers, one at each internal node of a particular mode, plus one of half their strength at the closed end, shift only that particular mode, and none of the others". (Actually the mathematics show that other ones of the higher modes are affected also, but in the case to be dealt with here, involving the fifth mode, those modes are well above the playing range.)

Several other useful and suggestive theorems like this one come out of the cosine-transform mathematics of perturbation theory. I think that the analogous theorems in the wave-mechanical formulation of Smith and Daniell, if they exist at all, might be very difficult to derive, and this is one of the reasons I prefer the simpler cosine-transform mathematics.

SLIDE 6

Slide 6 shows experimental results - "before and after measurements" - on a trumpet whose original fifth mode was badly depressed. In accordance with the single mode theorem, perturbers were placed at each of the four internal nodes of the fifth mode, the exact positions being located experimentally.

The raising of the fifth mode was not accomplished without a little disturbance of the neighboring modes, but you will note that the uncalled-for movements of the neighboring modes are only about five musical cents, about the limit of detectability by a human musician. So in one sense, at least, the results may be said to be perfect, for musical purposes.

CLOSING REMARKS

I should like to close with a note of caution that may admittedly contain some biased opinion:

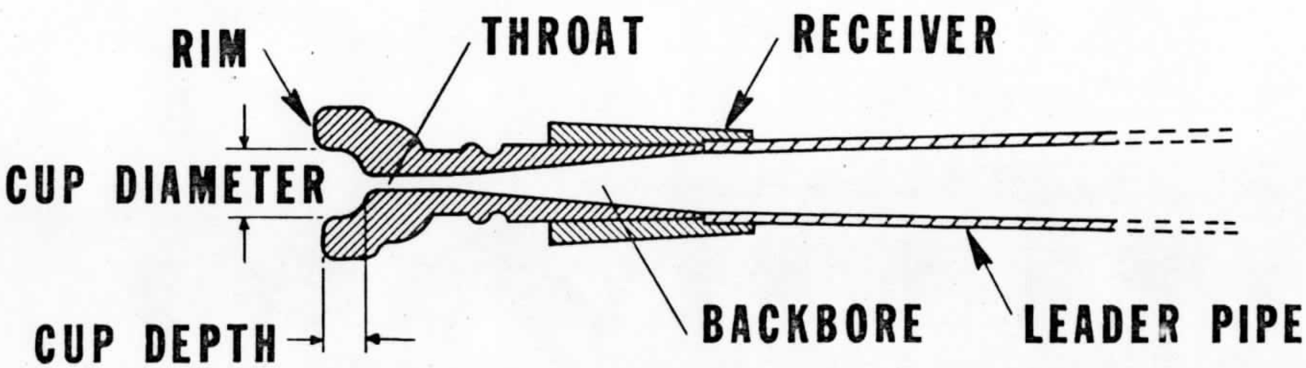
Smith and Daniell said in the abstract at the head of their article (quote) "a modified bore shape is calculated to give any desired correction in intonation" (unquote). To this sentence I should like to add: "*if* one is willing to restrain his correctional desires".

Some of the trumpets I have tested are so badly out of tune that their bores need more than just to be perturbed; they need to be completely undone, and then redone.

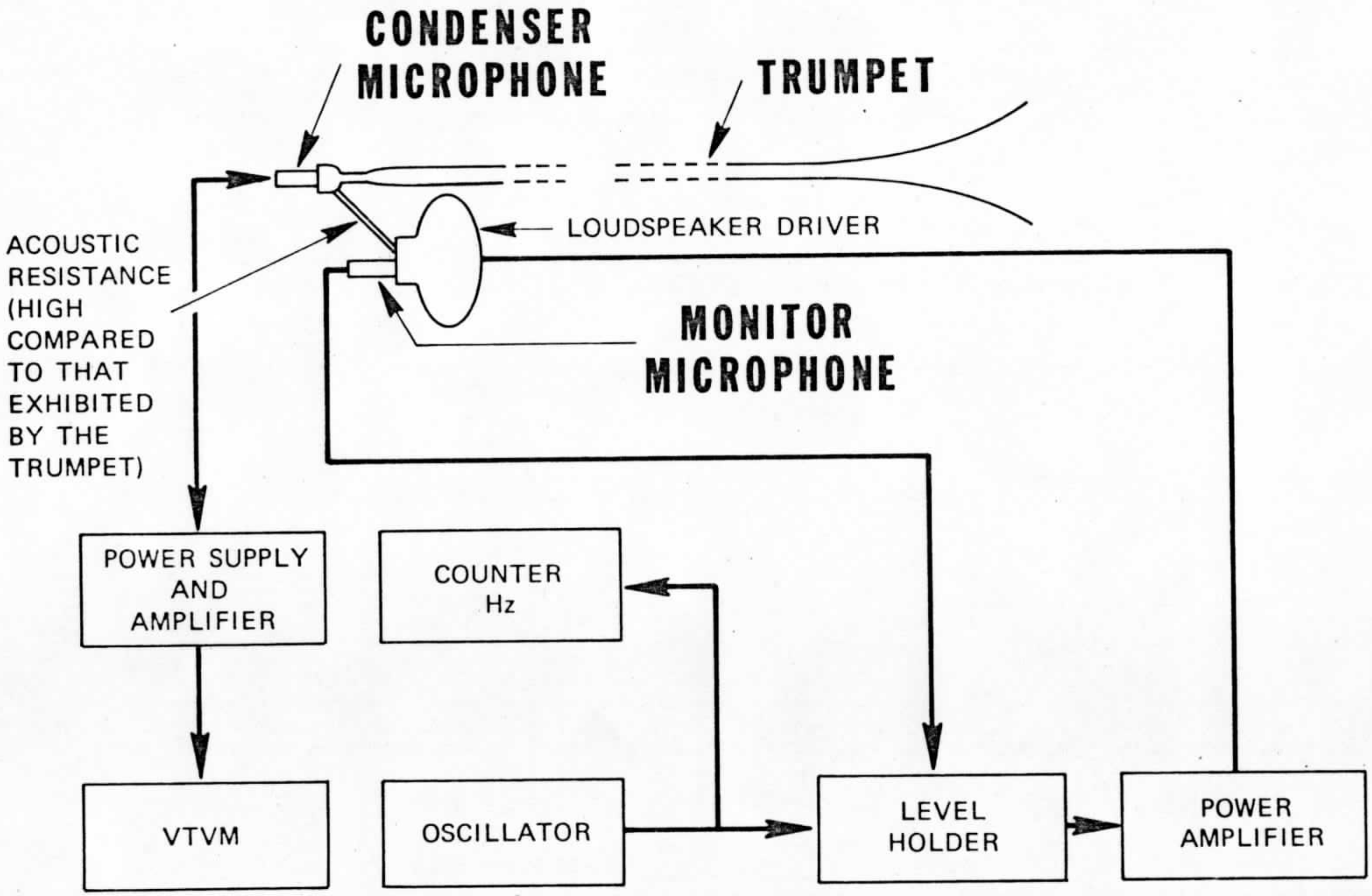
We have known, at least for the last ten years, how to design a new trumpet in any given key, having any given valve bore, - a trumpet that will have excellent intonation. Because we know how to do that, it is easier - and it seems to me, better - to do that rather than to correct a very bad trumpet.

However, for corrections no larger than a few tens of musical cents, perturbation methods should prove very useful.

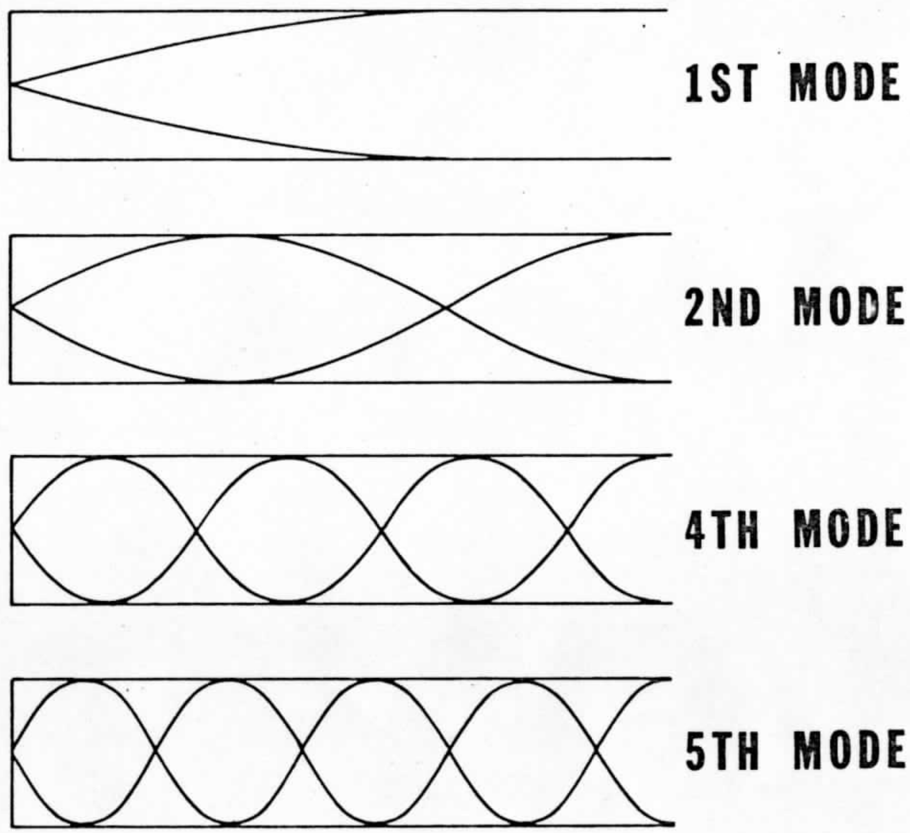

MOUTHPIECE
UNFLARED PART
BELL


RIM
THROAT
RECEIVER
CUP DIAMETER
CUP DEPTH
BACKBORE
LEADER PIPE

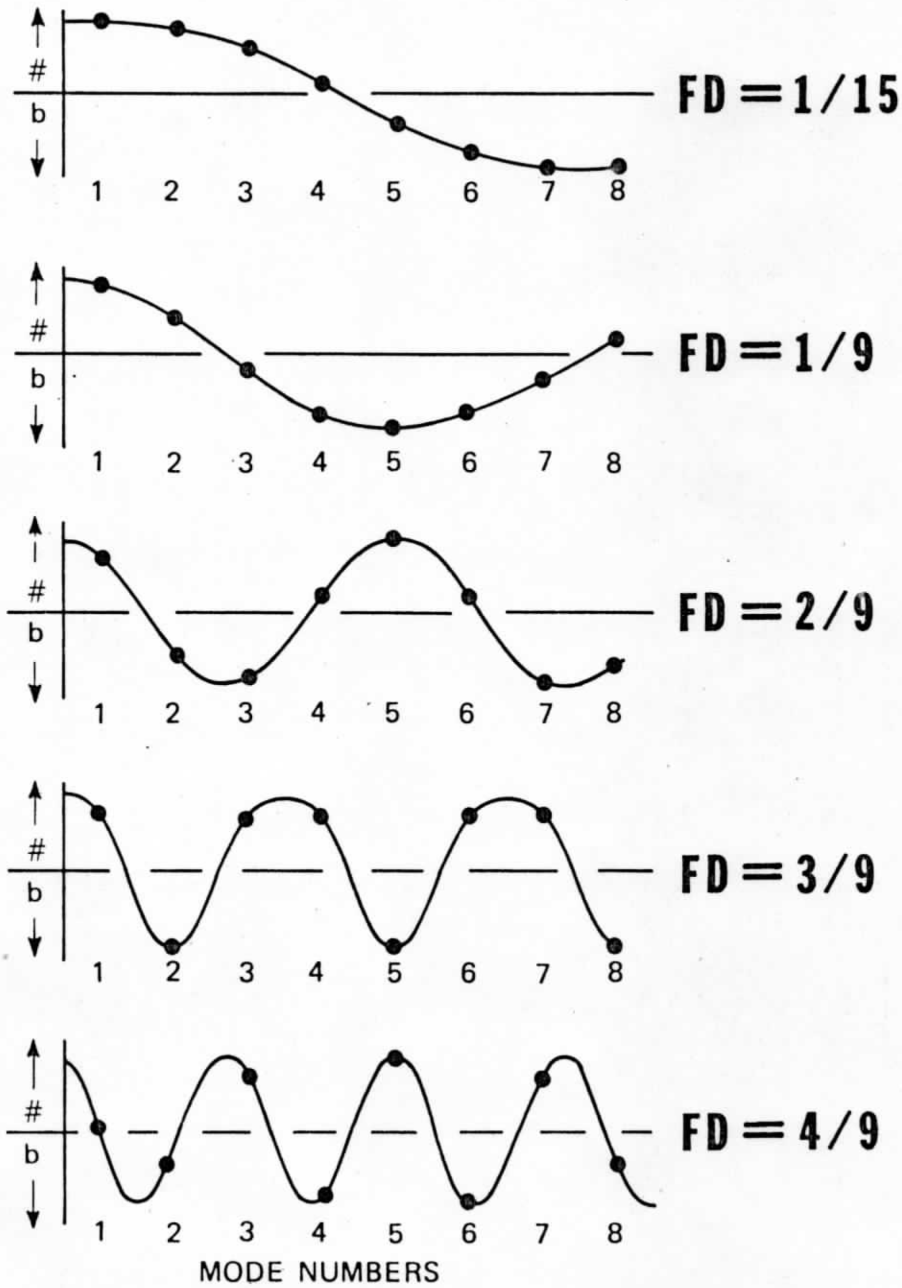
SLIDE 1


CONDENSER
MICROPHONE
TRUMPET
LOUDSPEAKER DRIVER
ACOUSTIC
RESISTANCE
(HIGH
COMPARED
TO THAT
EXHIBITED
BY THE
TRUMPET)
MONITOR
MICROPHONE
POWER SUPPLY
AND
AMPLIFIER
COUNTER
Hz
VTVM
OSCILLATOR
LEVEL
HOLDER
POWER
AMPLIFIER

SLIDE 2



SLIDE 3

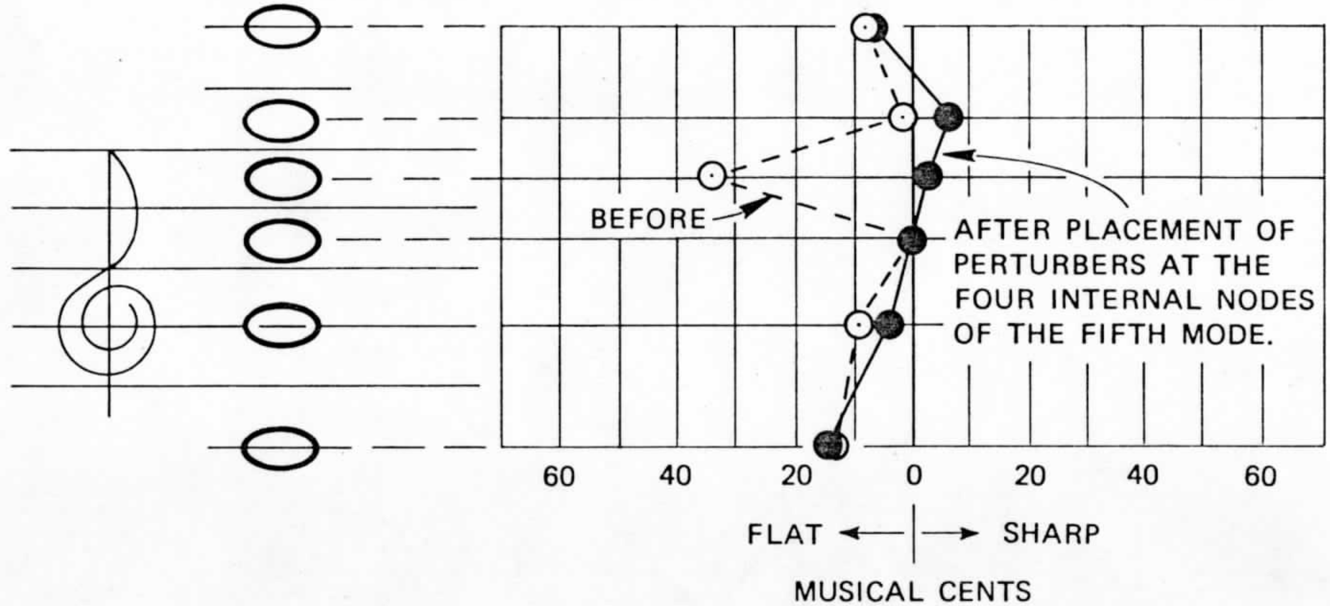


SLIDE 4

A SINGLE MODE THEOREM

EQUAL POINT PERTURBERS, ONE AT EACH INTERNAL NODE OF A PARTICULAR MODE, PLUS ONE OF HALF THEIR STRENGTH AT THE CLOSED END, SHIFT ONLY THAT PARTICULAR MODE, AND NONE OF THE OTHERS.

$$\frac{1}{2} + \sum_{i = 2,4,\ldots}^{2M-2} \cos \frac{(2m-1)\pi i}{2M-1} = \begin{cases} M-\frac{1}{2}, & m = M \\ 0 \text{ if } m \neq M \end{cases}$$



BEFORE
AFTER PLACEMENT OF
PERTURBERS AT THE
FOUR INTERNAL NODES
OF THE FIFTH MODE.
60
40
20
0
20
40
60
FLAT
SHARP
MUSICAL CENTS

SLIDE 6