TRUMPET INTONATION ACOUSTICS

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In the following discussion, we will be concerned with what we mean when we say a trumpet has "good intonation", and how intonation can best be determined, both by blowing tests and by electronic and acoustic measurements. We shall assume acquaintance with the fact that standard vibrational frequencies have long ago been agreed upon by (nearly all) musicians, and that the standard frequencies for all the other tones of the scale are referred to a standard "A" of exactly 440 Hertz (cycles per second).

Trumpeters who have learned to play exacting music on the modern valved trumpet are acquainted with the adjustments they must make in the length of the third valve crook when they play the written notes just below the staff. Trumpets of good quality often have an adjusting trigger on both the first and third valve crooks. So it is well known that some of the notes just at the bottom of the staff, and just below the staff, that are played with the same fingerings as notes up in the staff, are not in relative tune with those upper notes, unless "trigger" adjustments are made. Scrupulous trumpeters know also that even some of the unvalved, open tones are not in tune with each other. In fact, it is not uncommon with even the best of trumpets, for written e" to be noticeably flat with respect to written c", the tuning note. For this reason, exacting players sometimes play written e" as a valved tone (1-2) rather than as an open tone.

Fortunately for their listeners, most well practiced trumpeters can and do make lip corrections so skillfully that the imperfections of their instruments are not detectable by the listeners. In fact, a highly-skilled trumpeter may make his lip corrections so automatically, and so subconsciously, that unless he breaks habit, and concentrates consciously on sensing the tuning imperfections of the trumpet he is playing, he may himself be unaware of those imperfections. We may say, however, that a discriminating trumpeter, by definition, will become acquainted with the intonation imperfections of his trumpet, and will consciously work out the optimum types of correction, which then by practice will become automatic and subconscious with him.

Dale[C1] has given a good discussion of intonation imperfections, and of how the trumpeter should learn the intonation peculiarities of each of his instruments in order best to compensate for those peculiarities.

Dale has also shown graphically some typical intonation patterns, e.g. with the note e" flat, and note g" less flat, or even a little sharp. He did not say how his patterns were determined, but presumably they were determined by careful blowing tests.

The author uses electronic and acoustic apparatus that quantitatively measures the intonation errors of the trumpet alone, without the trumpeter blowing it. The apparatus will be described below, but first, a word may be in order for the trumpeter whose devotion to his own art has not afforded him a diversion into experimental and theoretical acoustics. To him it may seem unaesthetic, or worse, - unbelievable - that an assembly of electronic and acoustical apparatus could make a reliable test of the noble trumpet. Let him be confirmed in the belief that some of the properties of the trumpet are recognized to be beyond present quantitative description and translation into physical tests. However, the property of intonation is not one of these. Modern acoustic instruments permits intonation measurements with an accuracy and reproducibility that the human himself cannot approach. (For example, it is easy to show with the measuring apparatus that the human himself cannot blow exactly the same note in a set of successive tests.) The most important point however, is that human blowing tests and acoustic measurements do not disagree with each other. Many skilled trumpeters have visited the author's laboratory, and none of them has ever found the machine tests of his trumpet inconsistent with his own previous blowing tests. On the other hand, most of those trumpeters have expressed pleasure at having their own beliefs - and sometimes mere suspicions - confirmed, and put into confident and provable numbers.

It may be well to digress a bit here to specify exactly what we mean when we say none of those trumpeters has ever found the machine tests inconsistent with his own blowing tests, because in a later discussion, Professor Benade will say that "For many years acousticians were puzzled and frustrated because their measurements of the natural frequencies in wind instrument air columns did not correlate very well with the pitches played by musicians on these instruments". Professor Benade will then go on to explain how some of the higher natural frequencies of the wind instruments modify the pitches that would tend to be produced by lower natural frequencies when the instruments are blown. But the effects Professor Benade will be discussing are more subtle, both qualitatively and quantitatively, than those we are discussing here. Furthermore, as he will explain, they tend to operate principally (1) when the trumpet is in the low playing range (g♭ to c") and (2) when the trumpet is being played loudly. The effects we will be discussing here are not so subtle. When we measure the natural frequencies of the existing commercial trumpets, we do not need to wonder why musicians do not blow those same frequencies when they play; if they did they would not be good musicians.

What we mean here when we say the machine tests are consistent with blowing tests is that, for example, if the machine shows that when the tuning note, c", is set correctly, the next open note, e", is a third of a semitone flat, the trumpeter will always confirm that he has been "lipping-up" the e". Or, for another example, if we help a trumpeter determine the correct third-valve trigger-pull to put his d♭' in tune, he will always confirm that the trigger-pull we have indicated is either the same as what he has been using, or else it seems more comfortable, that is, it takes less corrective lipping than the pull he has been using.

Figure 1 represents the author's apparatus schematically, and Figures 2 and 3 are photographs of the actual equipment in the author's laboratory. The apparatus is of a generic type first described by Webster[C14] of C.G. Conn Ltd. Variants of it have continued to be developed and used at Conn, by E.L. Kent and his co-workers, and by Professor A.H. Benade at Case Western Reserve University. The particular apparatus used by the present author is patterned most closely after one used at the University of Southern California by Professor John Backus. Still other types of intonation-measuring apparatus, or more precisely, apparatus to measure acoustic impedances as a function of frequency, are mentioned and diagrammed in a recent Scientific American article by Professor Benade[C15].

Referring again to Figure 1: A trumpet (or sometimes only part of a trumpet, e.g. only a mouthpiece) has a constant, oscillatory current of air injected into it by a loudspeaker driver feeding through a high acoustic resistance (over twenty times the highest acoustic resistance ever exhibited by the conventional trumpet). The oscillatory air current is kept constant by a feedback loop, comprising a monitor microphone and a level holder, which controls the input to the power amplifier that actuates the speaker driver. The pressures produced in the mouthpiece are sensed by a microphone inserted into the mouthpiece. In the present author's apparatus the insertion distance is controllable by a micrometer adjustment so that different lip insertions used by different players can be simulated.

The essential results given by the tests are the values of the pressure sensed by the mouthpiece microphone, at the various frequencies.

Figure 4 shows typical resonance curves made with the described apparatus. The horizontal coordinates are frequencies in Hertz (cycles per second). They run from zero to 1700 Hz. (The highest trumpet note in Bach's Brandenburg Concerto No. 2 is a concert G at 1568 Hz). The vertical coordinates here are millivolts, as read from the vacuum tube voltmeter of Figure 1. The bottom curve represents a conventional B-flat trumpet; the middle curve, a high F trumpet; and the top curve a high B-flat (piccolo) trumpet.

The frequencies of the peaks in Figure 4 are the resonance frequencies, the so-called natural frequencies, at which the trumpet itself tends to cause the trumpeter to play. So the frequency positions of those peaks are the primary data of interest in this discussion. However, there are other types of information deducible from curves such as those of Figure 4. For instance, the sharpnesses of the peaks, and their distinctions from the surrounding valleys, are measures of the pitch selectivity of the tested trumpet. Or putting it the other way around, the breadths of the peaks determine how easily the trumpeter can "bend" the notes whose pitch he desires to shade, either because they are in error in the first place, or because the music being played requires pitch shading.

Comparison of the top curve with the bottom curve shows one reason why a piccolo trumpet is preferred for very high-pitched trumpet music. It is easier to land on any one intended resonance because the nearest neighbors are farther away. (The advantage is not, as those who have not played the piccolo trumpet might suppose, that it makes it easier "to get up there"). The main reason for the presentation of Figure 4 here is to indicate to the playing trumpeter that the pitches he selects with his highly developed lip control and breath control are definitely related to measurable physical properties of the instrument itself. However, the immediate concern here is only where the resonance peaks are on the frequency scale. How accurate are the pitches they represent?

If one plots only the frequencies of the peak amplitudes from curves like those in Figure 4 on a musical graph, one gets results like those of Figure 5, in which leftward displacements from the center vertical line represent flatness, and rightward displacements represent sharpness, both measured in musical cents, or hundredths of a semitone. Figure 5 shows intonation graphs for four modern B-flat and C trumpets. Each of the examples is from a highly respected European or American manufacturer, and each is a playing trumpet of a trumpeter who is very particular about what he plays. Obviously, however, the intonation of each of these trumpets is imperfect, and that fact was already suspected by the owners of the instruments before the tests.

The intonation patterns of Figure 5 all show flatting as the pitch rises, and that is not necessarily bad. Elaborate measurements in which the condenser microphone is moved gradually into the mouthpiece as the frequency is raised, show that the player can compensate for this flatting by gradually pushing more lip flesh into the mouthpiece as he goes toward the higher register. This action is quite natural. ~~and indeed, would be hard to avoid~~. On the other hand, it is intuitively obvious that zigzags in the intonation curve are objectionable, because they cannot be corrected by any type of smooth, monotonic action on the part of the player. The most commonly occuring fault is the leftward zig at the fifth mode, e" ~~(fourth space E)~~ followed by a rightward zag at the sixth mode, g". ~~(G just above the staff)~~. The fifth mode of the trumpet is one that professional players often compensate by alternate fingering when the tone is to be sustained and prominent.

To see how well players compensate for the tendencies of their instruments, we may look at Figure 6, containing an intonation graph for a B-flat trumpet owned by Edward Haug of the San Francisco Symphony. The points connected by the dashed lines show how the trumpet itself tends to play. When the fourth mode tuning note, c", is set at 466 Hz, the lower modes are a little sharp, but not seriously so. The fifth mode, e", is more than a third of a semi-tone flat, the sixth mode, g", is back in perfect tune with the c", and the eighth mode, c"' (high C) is about a quarter tone flat.

The points connected by the solid line showed what happened when Edward Haug blew the same trumpet, first putting the tuning note, c", exactly in place, and then blowing the lower and upper modes with only his ear to guide him. The lower modes were played at very close to the frequencies indicated by the measured resonances of the trumpet, but the fifth and eighth modes, written e", and written c"', were remarkably compensated, in fact overcompensated.

If the frequency errors of Edward Haug's blown notes, as represented in Figure 6, are averaged, and then for each mode, one considers the "relative error" to be the deviation from the average error, it turns out that the relative errors are so small as to be hardly detectable to musical ears. (The minimum detectable error is about [redacted] 5 cents.[redacted]) So Edward Haug had no difficulty in blowing correct notes from the trumpet in question, but he had to perform a zig-zag type of compensation, a large compensation at written e", no compensation at written g", and a large compensation again at written c'". There is no question about his ability to do this, but there is a question as to whether he should have to expend the effort, consciously, or unconsciously.

What causes the intonation zigzags? Can a trumpet be built that has no intonation zigzags (or only negligibly small ones?) A complete answer to the first of these questions cannot be given without the use of advanced mathematics, but a set of approximate answers can be given that may satisfy anyone who is not actually undertaking intonation correction of existing trumpets or the designing of new ones. The answer to the second question - can a trumpet be built that has only negligible intonation zigzags - is, happily, yes. [redacted]

One of the answers to the question of what causes the intonation zigzags pertains only to the fifth mode (written e"). Fourteen cents of the flatness of that mode is due to the fact that the trumpet at least tends to play natural harmonics whose frequencies are simple integer multiples of a fundamental frequency. On a "natural" (diatonic) scale, written e" would have exactly five-fourths the frequency of written c", the tuning note, but in the scale of equal temperament, used in modern music, written e", by definition, has a frequency 14 musical cents sharper than this "just major third". So even a perfect "natural trumpet" would play the modern written e" 14 cents flat. From a trumpet-lover's point of view, we might therefore always forgive at least 14 cents of fifth mode flatness, and indeed go on to complain about the compromises that had to be made to define and use the even-tempered scale. However, modern trumpets err worse than by being only "natural". The fifth mode is almost always found to be significantly more than 14 cents flat.

The subnatural fifth mode, and all the other zigzags, arise from undesirable variations in the shape of the trumpet air column.

At this point, it might seem most logical to examine in detail just what are the shapes of trumpet air columns, and then mention whatever undesirable variations in those shapes may occur. However, the over-all question of what makes intonation zigzags would then split up into various subsidiary questions for the different parts of the trumpet having different functions, and the complications would tend to obscure the main principles. For present purposes the over-all question of what makes intonation zigzags can be treated much more simply by imagining temporarily that the trumpet is a very simple kind of acoustic resonator, a pipe of uniform cross-section, closed at one end, and open at the other. Although the trumpet is far from such a simple resonator, the trumpet may be said to simulate such a resonator in some respects. At the bell end of the sounding trumpet there is always a velocity antinode (or pressure node) just as there is at the end of an open pipe, and inside the sounding trumpet there are alternate nodes and antinodes. Although the nodes and antinodes are not evenly spaced along the axis in the trumpet air column, particularly when the low modes are being sounded, they tend to become evenly spaced for the higher modes, just as are the nodes and antinodes in a uniform pipe. Some complicated differences between the air vibrations in a simple closed-open pipe and those in a trumpet air column occur at the mouthpiece end. There, if the end condition is thought about as if it were a velocity node, it must be thought about as if that node were traveling away from the middle of the trumpet as the frequency rises. However, if we recognize that we are making some simplifications just for thinking purposes, and if we check experimentally any important deductions we make from our simple model, there is no harm, and indeed there is considerable benefit in using the closed-open pipe model.

Now, it has been known for decades that if one makes a bulge (bore enlargement) in a tubular resonator, one will raise the frequency of all the vibrational modes that have a velocity antinode (pressure node) at the position of that bulge. But the same bulge will lower the frequency of all the vibrational modes that have a velocity node (pressure antinode) at the position of that bulge. A constriction (bore contraction) has just the opposite effects. It lowers the frequency of all the vibrational modes that have a velocity antinode at the position of the constriction, and it raises the frequency of all vibrational modes that have a velocity node at the position of the constriction. For present purposes, these points are best accepted as experimental facts that we can verify by bulging, or constricting, a tubular resonator. Highly sophisticated derivations of quantitative formulas concerning these effects are given in the articles of Schroeder and Mermelstein[C9,C10]. Qualitative statements of the facts are familiar in some of the musical literature, e.g. in Bate's book[C11]. In fact, Bate almost goes on to give the precaution we will give below when he says: "We must consider also that an irregularity in the bore which is near an antinode when the fundamental is sounding may be approached by a node when the air column breaks up to give a harmonic. In this way it is possible for some harmonics in a given series to be out of tune with their prime tone as well as mathematically incommensurate with each other----".

Let us now refer to Figure 7 in which are represented the standing waves that could occur in a simple tubular resonator closed at the left end and open at the right end. Six different modes are represented, the first and second, the fourth,fifth, and sixth, and the eighth. The velocity antinodes (regions of maximum motion) are represented by the spread-apart maxima and minima of the sinusoidal curves, and the velocity nodes (regions of zero motion but maximum pressure variation) are represented by the crossings of the sinusoids. For each mode, a velocity antinode exists at the right end of the resonator and a velocity node at the left end. The total number of nodes is always equal to the mode number.

Now, looking at Figure 7, let us consider what would be the effect of a constriction placed at the right end of the tube. Every one of the modes has a velocity antinode at the right end, so according to the rules already mentioned, every one of the modes would be lowered (or flatted).

Next, let us consider what would be the effect of a constriction placed at the left end of the tube. Every one of the modes has a velocity node at the left end, so according to the rules already mentioned, everyone of the modes would be raised (or sharped).

Next, let us consider what would be the effect of a constriction placed just slightly away from the left of the tube; let us say, one-fifteenth of the total length away from the left end. Now we notice that for the lower modes (one and two) the constriction is still nearer to a node than to an antinode, so those modes will be sharp, but by the time the fifth mode is reached, the constriction is slightly nearer to an antinode, so now it is beginning to cause the modes to be lowered in frequency rather than raised, and when the eighth mode is reached, the constriction is exactly at an antinode and produces its maximum amount of frequency lowering. Note here that in the progression from the first to the eighth modes the constriction at one-fifteenth of the total length from the left end has only a monotonic effect. It sharps the first mode the most, sharps the second, third and fourth modes successively less, then begins to flat the modes, and flats them successively more and more (at least up to the eighth). However, if we now place the constriction any farther in from the left end, we will begin to get not a monotonic, but rather an oscillating, or zigzag effect.

For instance, let us place it at one-ninth of the total length from the left end. If we examine this position with respect to the nodes and antinodes of the various modes, we will deduce that there will be sharping of the first and second modes, but by the time the ~~fourth~~ third mode is reached there will be flatting. There will be maximum flatting at the fifth mode, after which the flatting will diminish. By the time the eighth mode is reached there will be a slight sharping again. So now we have produced a slow flatting zig (or less and less sharping zig) from the first to the fifth modes, and then a slow sharping zag (or less and less flatting zag) from the fifth through the eighth modes.

All of the deductions we have been making in the last few paragraphs can be summarized in diagrams like those of Figure 8, in which mode shifts (flatting or sharping) are plotted against mode numbers. In Figure 8, each of the diagrams is for a different fractional distance of the constriction from the left (closed) end of the tube. (The particular fractional distances chosen for the second to fifth diagrams of Figure 8; 1/9, 2/9, etc., were chosen because those fractional distances mark either nodes or antinodes of the fifth mode((written e")) and later in this discussion, particular attention is going to be drawn to those positions in connection with actual tuning of the fifth mode.)

The observations we make from Figure 8 are that a constriction in the closed-open tube very near the closed end causes a slow flatting as we go from the first to the eighth modes (one zig), but as the fractional distance of the constriction from the closed end is increased, the mode shifts become wavelike, first flatting, and then sharping (zigzags). The waves grow shorter as the fractional distance grows longer. (As a matter of mathematical fact, the lengths of these waves, measured in mode numbers, would be exactly equal to the reciprocal of the fractional distance of the constriction from the left end, and the first maximum of the wave would always be at the fictitious mode number, 1/2. We can see, for example that the wave for the fraction 3/9=1/3 has a wavelength of three mode numbers, so it has maxima at 1/2, 3-1/2, 6-1/2, etc.)

In Figure 8 we have not shown a diagram for a constriction at the fractional distance one-half, because we wanted to hold down the number of diagrams, and that particular diagram is so easy to describe in words. In that diagram the wave on which the modes seem to ride would be exactly two mode numbers from crest-to-crest, or one mode number from crest-to-trough, and the first half-wave would go from a high at 1/2 to a low at 1-1/2, leaving the mode number 1, exactly in the middle, neither sharped nor flatted. Similar statements, mutatis mutandis, would apply to each of the other modes. The wave would leave each of them exactly between a crest and a trough, neither sharped nor flatted.

The one-half position, exactly midway between the closed end and the open end, has the very special property that a constriction there would produce no shifting of any mode; and as we have already learned, because an enlargement produces an effect just the negative of that produced by a contraction, and the negative of zero is just zero, neither would an enlargement produce a shifting effect on any of the modes. This tells us one possible reason why trumpet valves should be placed approximately at the center of the air column of the trumpet, as they usually are. To the extent that a trumpet behaves like a simple closed-open tubular resonator, its modes should be undisturbed by constrictions or enlargements at its middle, so if the valve design does not maintain a constant cross-section, the modes should still not be greatly disturbed. This general idea is good, but it works imperfectly, mainly because the trumpet does not behave exactly like a simple closed-open resonator; its effective acoustic center changes as the modes change, and as we can see from the last diagram in Figure 8, if a constriction is only slightly displaced from the center its effect is maximally oscillatory among the modes, one mode being sharped, the next being flatted, the next being sharped, etc. When we see zigzag intonation such as that of Figure 5D we can suspect valve constriction (or enlargement) and sometimes, through experiments, we can prove that the zigzag comes from one or more of the valve passages.

Up to this point we have discussed the mode shifts caused by constrictions, or enlargements, in only one half of our acoustic resonator, the part from the closed end to the middle, corresponding, in the trumpet, to the part from the mouthpiece end-plane to the middle of the air column. What happens if the constriction is beyond the half-way point. In that case, we can show, using reasoning similar to that we have used above, that it is most convenient to specify the position of the constriction in terms of its fractional distance from the open end, instead of the closed end, so that the fractional distance will again be a number between zero and one-half. Then we can show that the effect of that constriction will be just the same size as the the effect of a constriction having the same fractional distance from the closed end, ~~except that it will be the negative of that effect.~~ but it will have the opposite sign. One effect will be the negative of the other. To illustrate, a constriction at the fractional distance 2/9 from the closed end will produce a maximal sharping of the fifth mode, as shown in the third diagram of Figure 8, but a constriction at the fractional distance 2/9 from the open end will produce a maximal flatting of the fifth mode. The diagram for the constriction at the fractional distance 2/9 from the open end would be just like the third diagram of Figure 8, except that it would be turned upside down.

With only the knowledge we have acquired here, we can picture possible constriction and enlargement configurations that will account for intonation curves of the types that actual trumpets exhibit. For instance, let us refer back to our most zigzagged actual example, D of Figure 5. It shows a mode-to-mode zigzag superimposed on a monotonic flatting curve. We have already discussed how zigzagging could come from a constriction in the neighborhood of the valves. If we now refer to the first diagram of Figure 8, we can see how a monotonic flatting could be produced, as we go from the second mode to the eighth. This could come from ~~too much~~ a constriction in the neighborhood of the mouthpiece - leaderpipe junction. However, we must not forget that an enlargement at the bell end acts similarly to a constriction at the mouthpiece end, so for the monotonic flatting it is just as logical to suspect the bell flare. (The use of the verb "suspect" is not meant to imply that we are necessarily looking for a culprit. We may indeed believe in monotonic flatting, from the second mode to the eighth, as being very natural for the player to compensate, in which case we ascribe no fault. Many highly esteemed, professional-quality trumpets show monotonic flatting. However, it is a strong belief of the present author that the best trumpets of the future will not show it.)

As of the time of this writing, a technical article has recently appeared, by W. Kruger, on methods of pitch correction of wind instruments[C16]. Kruger's teachings contain, explicitly, some of the mathematics that has underlain the statements and the illustrations of the present discussion. Kruger gives some examples of actual experimental tunings of instruments, but to follow his descriptions at all well, one needs to follow the helping mathematics. In verbal terms, such as we have been using here, one might say that Kruger thought and worked in terms of curves like those of Figure 7 and 8. Whatever kind of erroneous intonation wave the instrument had, he tried to produce the opposite, or correcting, kind of wave with one or two suitable changes in the bore. He was successful in improving the intonation of several instruments with which he worked. Anyone who wishes to go deeply into subject of intonation faults and their elimination would do well to study his article.

Kruger hinted at what could be done beyond what he actually did. It would be possible to use the mathematics of Fourier analysis much more comprehensively than he did, both to find, and to correct, intonation errors of faulty wind instruments.

Although, in a non-mathematical discussion such as the present one, it is hardly possible to advance the general concepts taught by Kruger, there is at least one very interesting theoretical and experimental question that he did not mention, which we can discuss.

The question: Is it possible to shift the pitch of only one mode at a time, without affecting any of the others? The answer is yes, and we shall show experimentally that a single mode can be shifted, more than a third of a semitone, without affecting the other modes by as much as could be detected in musical playing. It is an interesting exercise, but not something one would want to do for fun, after once showing it could be done, and almost certainly not something one could do for profit. To do it correctly is too time-consuming.

The preceding teachings have showed us that making any constriction at a single place in the trumpet affects all the modes, and produces intonation-change-waves like those shown in Figure 8. Now, we can show mathematically that if we put sets of constrictions in properly related places, so that they make sets of intonation-change-waves that have their crests and troughs in properly different places, we can make the waves cancel each other out at some of the mode numbers, whereas at other mode numbers they will reinforce each other. In fact, we can show that if we add a set of mode shift vs. mode number waves, like the ones shown in Figure 8, for the fractional positions 2/9, 4/9, 6/9 (the negative of the 3/9 wave) and 8/9 (the negative of the 1/9 wave) the four waves will add up to produce only a slight flatting, and exactly the same flatting, at all the modes except the fifth. However, at the fifth mode, all of the waves will reinforce each other and produce a strong sharping of that mode. Figure 9 shows the wave that is the sum of the four mentioned waves.

The general prescription for raising one mode at a time, and not affecting the relative intonation of the other modes, is to put constrictions at all the internal velocity nodes of the mode to be raised. By the word "internal" we are excluding the "node" at the mouthpiece end, which is unavailable to us for constriction purposes. (The general prescription for lowering one mode at a time, without affecting the relative intonation of the other modes, would be to put constrictions at all the internal velocity antinodes of the mode to be lowered.)

Figure 9, showing the theoretical effect of putting constrictions at the four internal nodes of the fifth mode, gives us considerable hope that we might be able to do single-mode-tuning of the fifth mode. Our hope has to be tempered by the fact that Figures 8 and 9 were derived from considerations of simple closed-open cylindrical resonators, and we know the trumpet is not so simple. There is actually some more complicated theory that we could use on this problem that would lead us to some longs complicated constriction shapes to compensate for the non-cylindrical air column of the trumpet, shapes that would still produce exactly the type of desirable effect shown by Figure 9. However, complicated constriction shapes are unappealing in advance. It is hard enough, experimentally, to handle simple, short, shapes of constant cross-section, particularly when they have to be bent and fastened into curved parts of the trumpet. So our practical hopes must be pinned on the trumpet being a near enough approximation to the simple, closed-open resonator; and it turns out that, for practical musical purposes, the approximation is good enough.

The trumpet chosen for the single-mode-tuning experiment was a C-trumpet, of generally professional quality, but specifically deficient in its fifth mode flatness. Figure 10 shows the intonation graph of the trumpet before the tuning, and gives an advance look at the result that was achieved by single-mode tuning.

The first experimental step was to find the positions of the four internal nodes of the fifth mode within the trumpet. We knew beforehand that these positions would not be at the fractional positions along the total length of the trumpet corresponding to the simple numbers, 2/9, 4/9, 6/9, and 8/9. Our best simple-theoretical guess was that the nodes would be at positions corresponding to those simple fractions times the "apparent length" of the trumpet at the fifth mode. (The "apparent length" at the fifth mode is equal to nine times the velocity of sound, divided by four times the frequency of the fifth mode. For this trumpet, at the temperature of measurement, it was 119 cm., whereas the total actual length of the trumpet was 124 cm.)

We determined the actual positions of the nodes, one-at-a-time, by putting in one constriction, a piece of brass rod about 3 cm. long(1/4 - inch (0.635 cm) rod in the "valve bore" locations, and 3/8 - inch (0.953 cm) rod in the 8/9, bell, location.) For each trial location the frequencies of all the modes, from the second through the eighth were measured, with the object of seeing how nearly the corresponding mode shift vs. mode number wave of Figure 8 could be approximated. The approximations were never inspiring, but the ways in which they deviated from Figure 8 curves were expectable from non-simple theory. The loops representing flatting were larger than the loops representing sharping, and the waves did not have constant periods. The very wiggly waves for positions near the "4/9 position" and the "6/9 position" were sometimes difficult to interpret at all. On the other hand, those very wiggly waves made the best locators for the nodal positions, because they were so sensitive to slight displacements of the constriction.

For each node, the position finally selected was the one in which the Figure-8-type wave showed not only an apparent maximum sharping for the fifth mode, but also symmetrical, equal, sharping (or flatting) for the neighboring fourth and sixth modes. It is believed that the finally determined positions were correct to within about one centimeter.

For the reader's possible interest, the final nodal positions, measured from the mouthpiece end-plane of the trumpet were: 25.4, 50.8, 80.0, and 106 cm.

After the nodal positions had been determined, final constriction sizes were calculated. Theory such as that found in the Kruger article[C16] tells us that to raise a frequency a third of a semi-tone, about 2 per cent, the total "apparent volume" of the constrictions should be about 2 per cent of the "apparent volume" of the air column. The "apparent volume" of the air column is equal to the apparent length, already defined, times the "valve bore" cross-section. Each constriction in a part of the trumpet that is of "valve bore" should, therefore, have a volume of about 1/2 per cent of the apparent volume of the air column. However, wherever the bore is larger, as it is at the 6/9 and 8/9 positions, the actual volume of the constriction should be enlarged by the ratio of the actual cross-sectional area to the "valve bore" cross-sectional area.

The actual final constrictions used in the experiments were: at the "2/9 position", a 1/8 inch (0.318 cm) rod 3.2 inches (8.13 cm) long; at the "4/9 position", the same; at the "6/9 position, several short pieces, flexibly strung together, of 1/8 inch (0.318 cm) rod, to make a total length of 3.6 inch (9.1 cm); and at the "8/9 position, a piece of

1/4 inch (0.635 cm) rod, 2.5 inches (6.35 cm) long. The constrictions were held in their places by small wire loops of negligible volume.

The final tuning results, as already mentioned, are shown in Figure 10. The fifth mode was fully corrected (up to mean temperament standard pitch). The second and eighth modes were hardly moved at all. The only significant departures from the ideality of single-mode-tuning were shown by the third and sixth modes, but even those departures were just at the borderline of musical detectability (about 5 cents).

So, we have now added to the general intonation improvement results shown, for example, in the article of Kruger, a demonstration that, with sufficient effort we can even tune only one mode at a time. This fact, standing alone, would seem to indicate that, with sufficient effort, using methods such as we have discussed, all the modes of a trumpet could be put into perfect alignment. This is true, with reservations. The main catch is that we have been discussing in these pages, only the usual nominal playing modes, the second through the eighth. Although it may not affect the other modes from the second through the eighth, the kind of tuning required to bring a single mode into near perfect alignment with its neighboring modes may throw out of alignment some of the upper modes above the eighth. We may now say that the quotation from Bate, already given, not only hinted how even single-mode-tuning might be accomplished, but forecasted accurately the kind of disadvantage it might have. It is well known that the characteristic trumpet tone depends upon the presence of many of the harmonics of the prime tone being sounded. Misalignment of harmonics must tend to darken the tone. That alone might not be undesired. It might even be desired, but if so, there would be better ways to bring it about. The most serious concern is that misalignment of upper harmonics deteriorates the response of an instrument as has been shown by Benade[C8,C15].

It seems not unreasonable to predict that, in the long run, our increasing knowledge of all the things we can, or could, do to correct imperfect intonation will bring us around to a modern, scientifically-supported, confirmation of the traditional belief mentioned by Bate[C11]: "If - we examine a large number [of trumpets], both antique and modern, we come to the conclusion that makers have almost always regarded sudden changes in the size of the bore as undesirable". We might extend this to cover changes in the slope of the bore, such as would occur at the junctions of conic frusta of differing taper.

It is possible to design a monotonically-smooth-bored trumpet to give any desired type of monotonically smooth intonation pattern, and that may come to be agreed upon as the best procedure.

As we approach the end of this particular discussion, we may appropriately take a long look backward in time, and at least a short look forward. Trumpets have been made and played for literally thousands of years, and music has been made upon them for at least a few hundred years, but until very recently, the shapes of the air columns inside trumpets have been changed only by trial and error.

The trial part is forever indispensable, but much of the error part can now be avoided. We are now learning the physical reasons for the various shape features of the trumpet air column, and we can not only explain why the old shapes worked (when they did) but we can also design new shapes that will work better than the old. We can already not only correct faulty intonation, but can design a trumpet ab initio that will have good intonation[C7].

We have not dealt here with the ever-present valve problems. Partly, this was so that we could devote more discussion to the single-mode-tuning problem, and partly it was because there are already good treatments of the valve problems in the literature[C2,C3,C12]. The article of Young[C5] even discusses the differences in valve crook lengths that might be allowed because the third valve crook is used less often and may therefore not be at the same temperature as the first and second valve crooks.

A final comment: We now know that the many years of evolution of the trumpet did not lead to intonation that we could call ideal, but ideal intonation (at least as we have so far defined it, among the first eight modes) is quite attainable if trumpeters demand it. The results of Benade and his co-workers, as reported for instance in the Scientific American article[C15], have shown that the upper resonances, above the eighth mode, have important cooperative effects with the lower resonances we have been discussing here. Improving the trumpet must involve improving those cooperative effects. But there is no barrier to improvement that has yet become apparent. There seem to be exciting possibilities ahead.

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C12 Ref. C4, pp. 10-13

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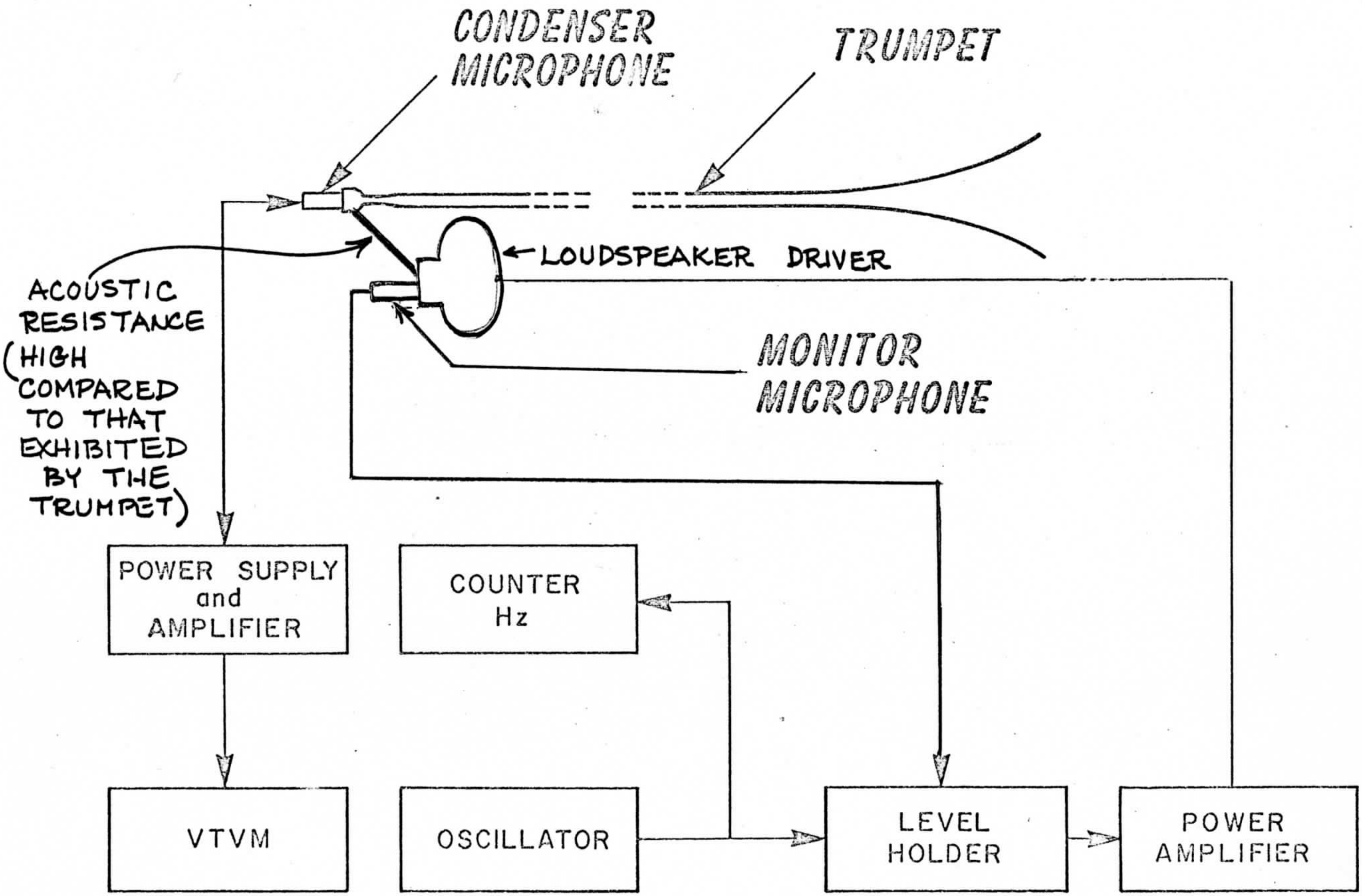

CONDENSER MICROPHONE
TRUMPET
LOUDSPEAKER DRIVER
ACOUSTIC RESISTANCE (HIGH COMPARED TO THAT EXHIBITED BY THE TRUMPET)
MONITOR MICROPHONE
POWER SUPPLY and AMPLIFIER
COUNTER Hz
VTVM
OSCILLATOR
LEVEL HOLDER
POWER AMPLIFIER

FIG. 1

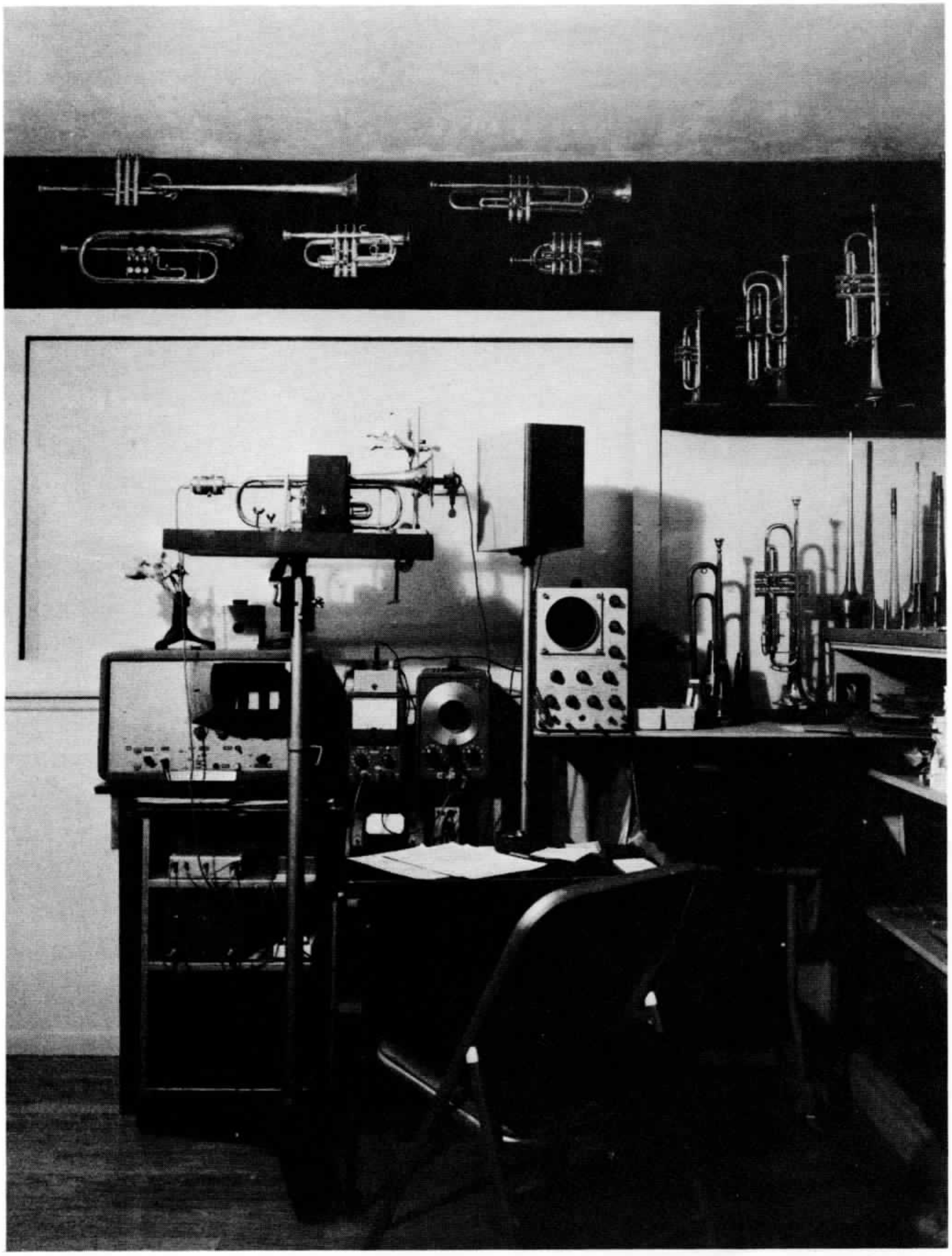


FIG. 2

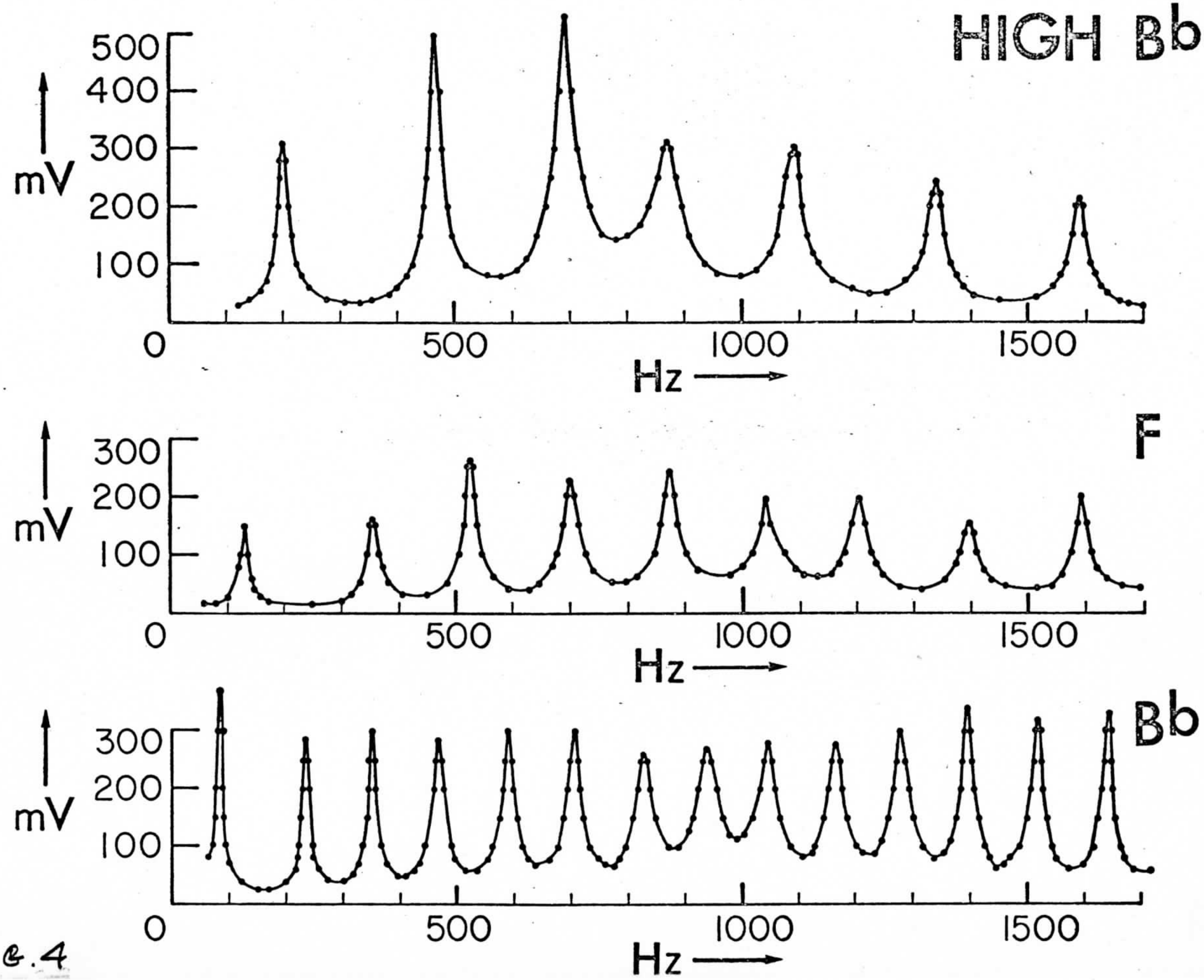

HIGH Bb
500
400
300
200
100
mV
0
500
1000
1500
Hz
F
300
200
100
mV
0
500
1000
1500
Hz
Bb
300
200
100
mV
0
500
1000
1500
Hz

FIG.4

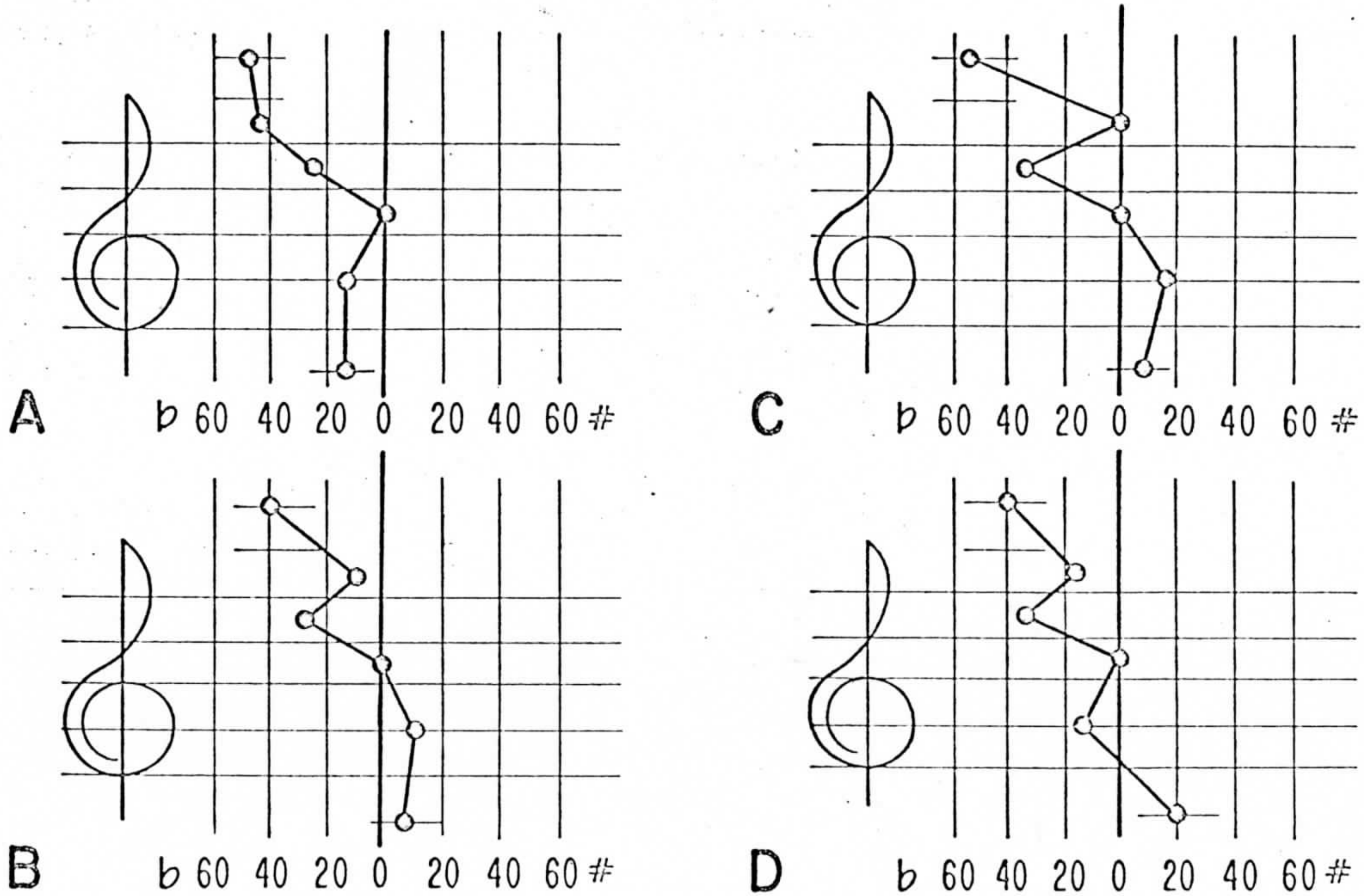
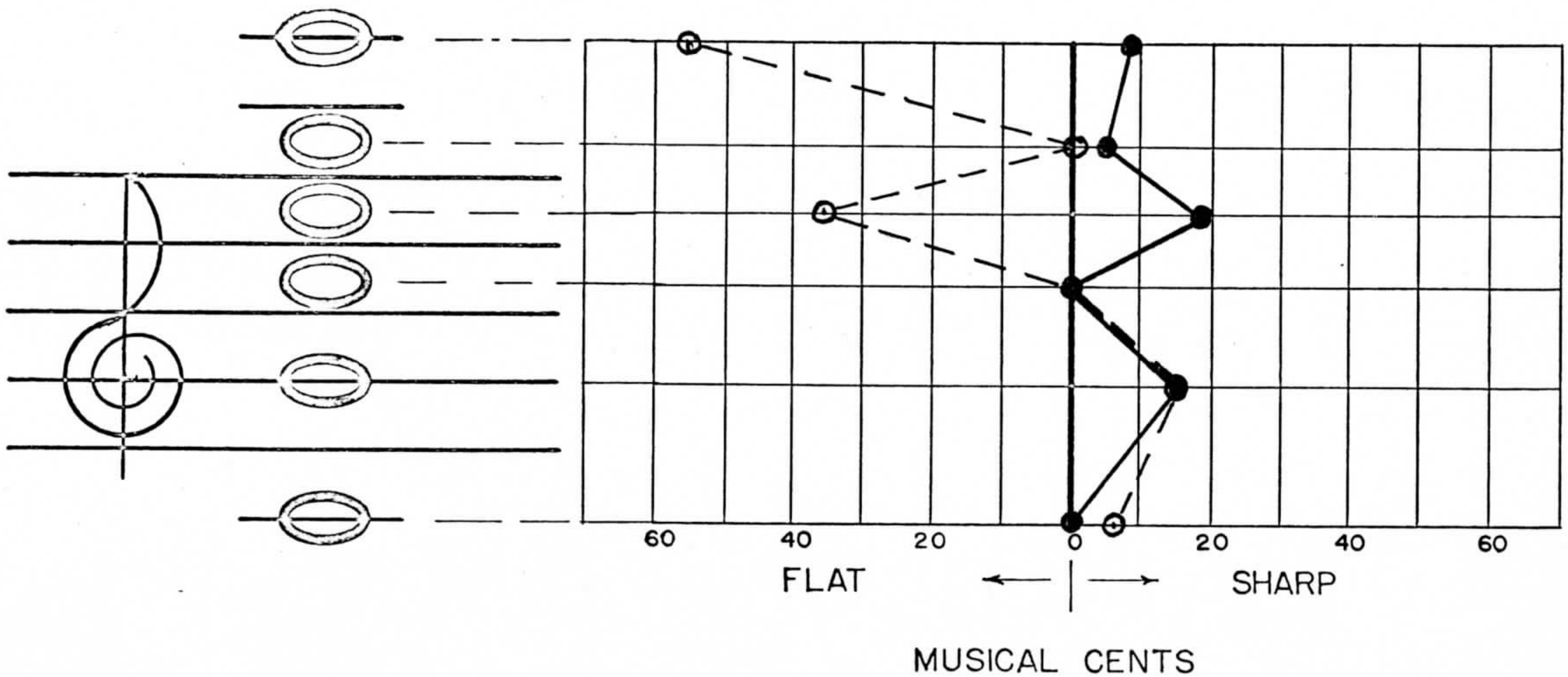

A
♭ 60 40 20 0 20 40 60 #
C
♭ 60 40 20 0 20 40 60 #
B
♭ 60 40 20 0 20 40 60 #
D
♭ 60 40 20 0 20 40 60 #

FIG. 5



TRUMPET INTONATION TEST

FIG. 6

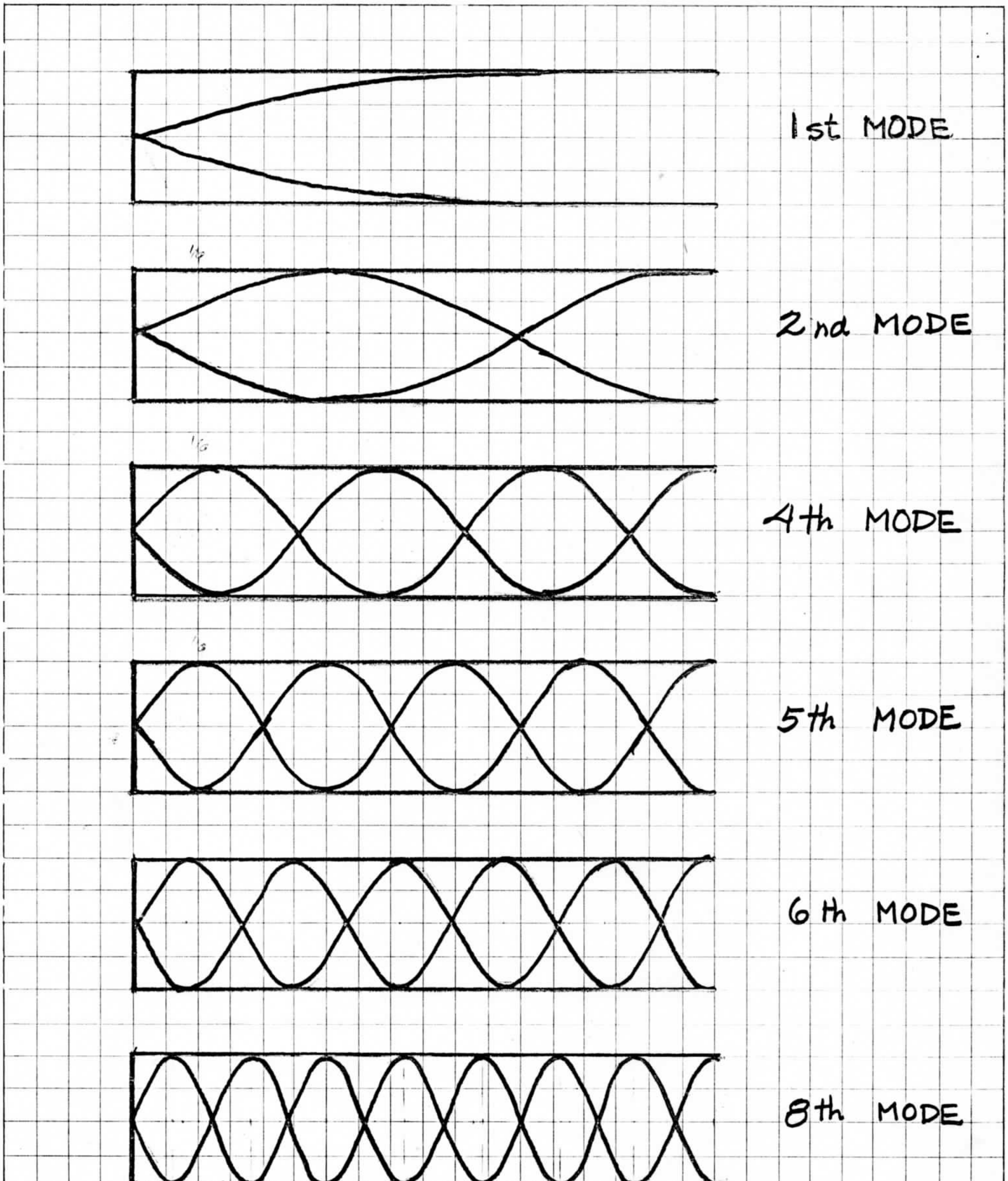
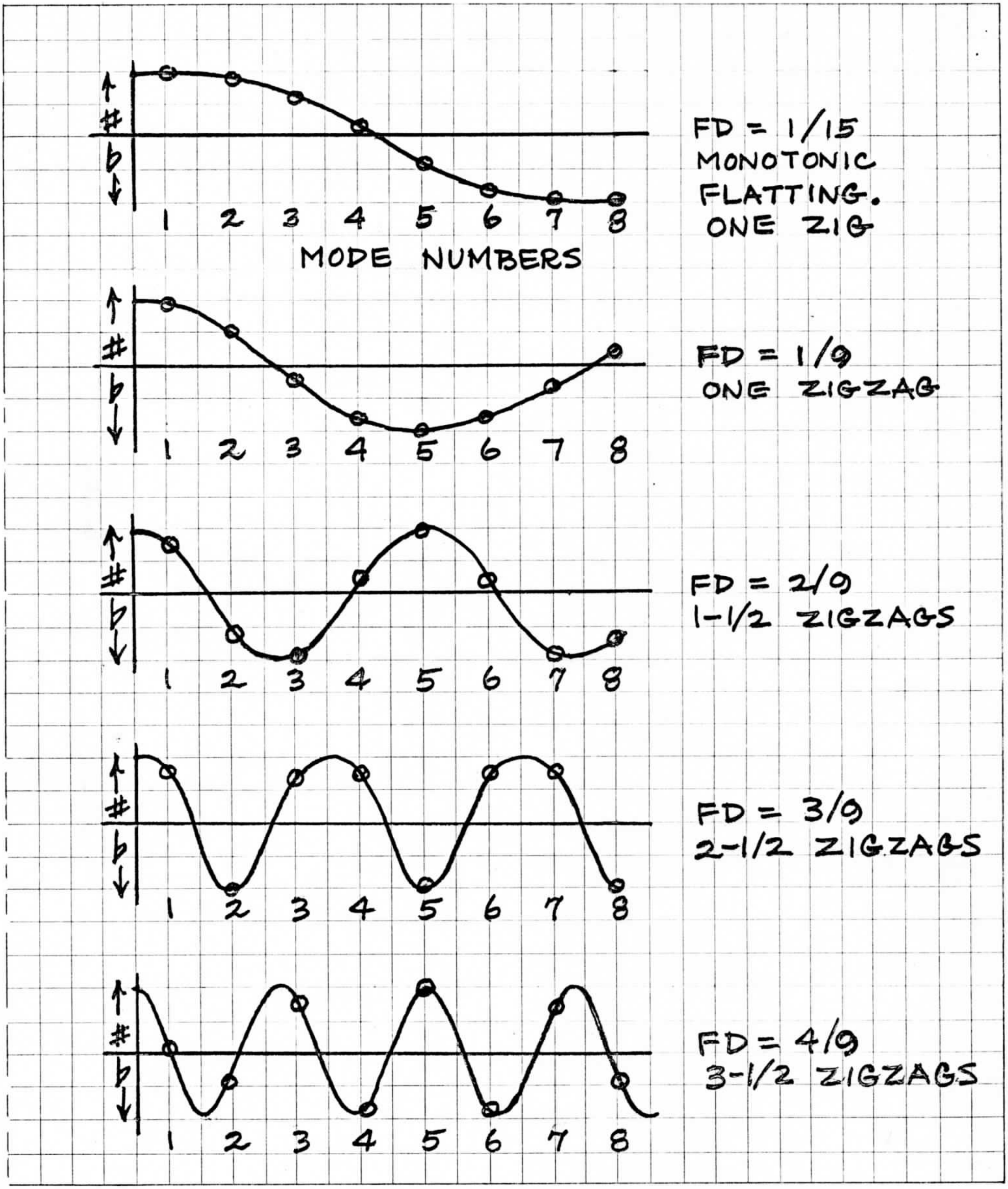


FIG. 7



↑ # b ↓
1 2 3 4 5 6 7 8
MODE NUMBERS
FD = 1/15
MONOTONIC
FLATTING.
ONE ZIG
FD = 1/9
ONE ZIGZAG
FD = 2/9
1-1/2 ZIGZAGS
FD = 3/9
2-1/2 ZIGZAGS
FD = 4/9
3-1/2 ZIGZAGS

FIG. 8

```
Z1,Z2,Z3,Z4,Z5
? 0.111111,0.222222,0.333333,0.444444,0
W1,W2,W3,W4,W5
? -2.5,2.5,-2.5,2.5,0
A,B,C
? 2,8,0.25
   2.0000 - 1.25
   2.2500 - 0.17
   2.5000 + 0.38
   2.7500 + 0.00
   3.0000 - 1.25
   3.2500 - 2.79
   3.5000 - 3.75
   3.7500 - 3.34
   4.0000 - 1.25
   4.2500 + 2.16
   4.5000 + 5.95
   4.7500 + 8.89
   5.0000 +10.00
   5.2500 + 8.89
   5.5000 + 5.95
   5.7500 + 2.17
   6.0000 - 1.25
   6.2500 - 3.34
   6.5000 - 3.75
   6.7500 - 2.79
   7.0000 - 1.25
   7.2500 - 0.00
   7.5000 + 0.38
   7.7500 - 0.17
   8.0000 - 1.25

PAUSE AT LINE  265
```

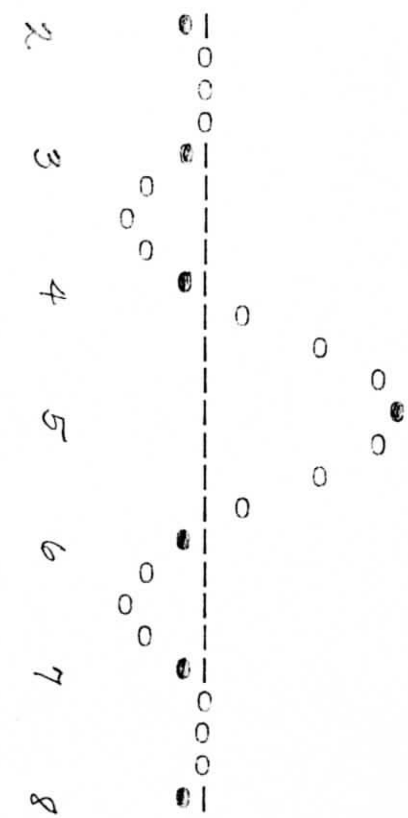


FIG. 9

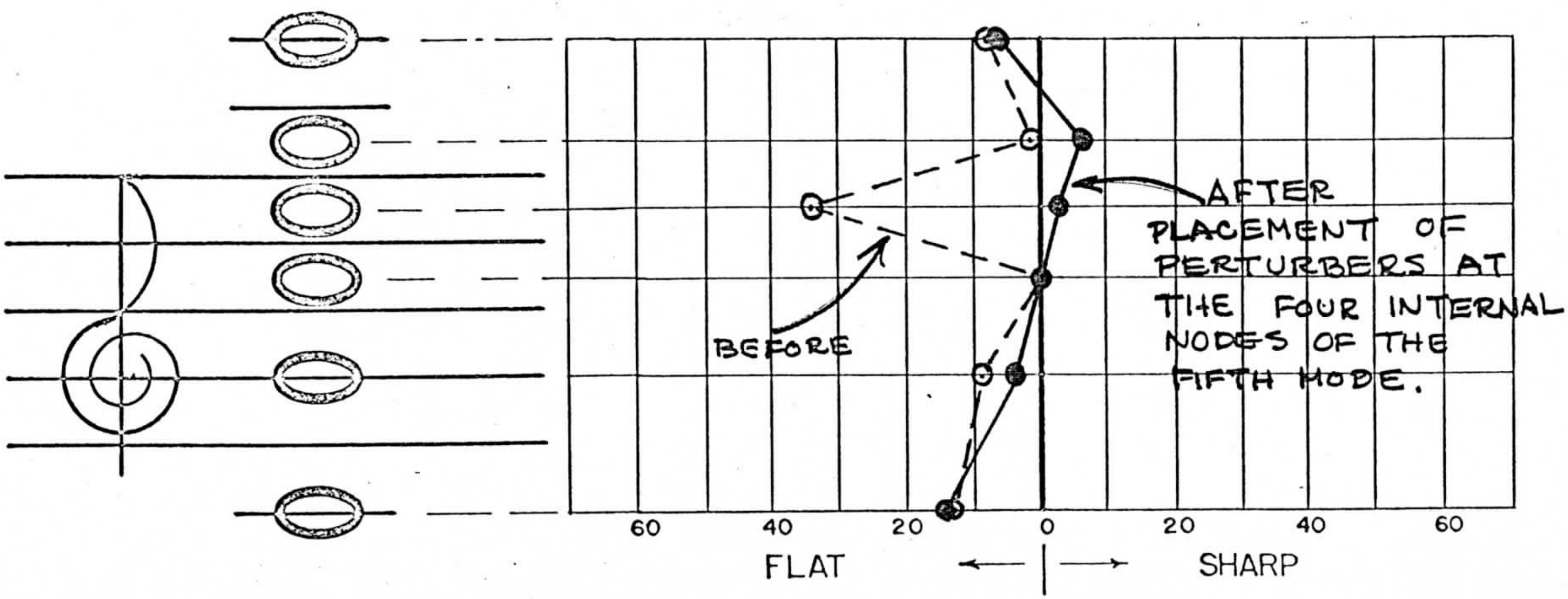


FIG. 10

TRUMPET INTONATION TEST

Olds C-Tp
8-26-73